Number Sequence Prediction Problems for Evaluating Computational Powers of Neural Networks

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MOTIVATION

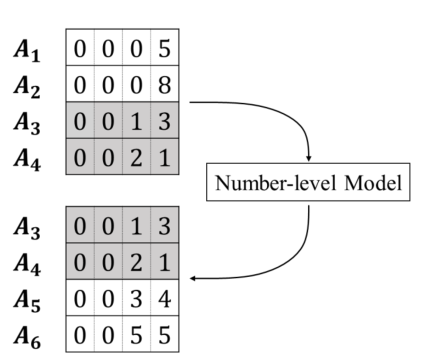
- Neural networks have been very successful for learning tasks on various data types including **image data and phonetic data**
- But it is hard to find well-defined **discrete and algorithmic** tasks where neural networks have been successfully applied
- Inspired by **number series tests for human intelligence**, number sequence prediction tasks can **assess computational powers** of neural networks

OBJECTIVES

- Define a set of algorithmic machine learning tasks with **numerical sequences**
- Quantify the **complexities** of simulating the sequence generation rules
- Evaluate **computational powers** of current deep learning models

TWO TYPES OF PROBLEMS

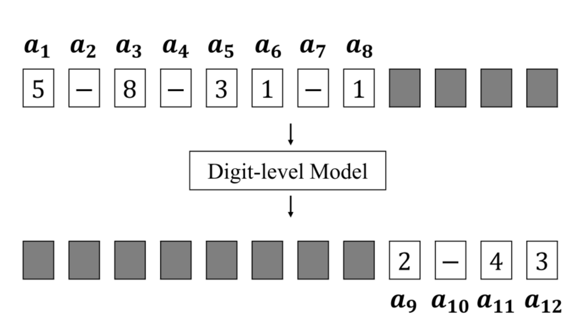
Number-level (CNN)



Digits in a 2-dimensional grid

Predict the **numbers in parallel**

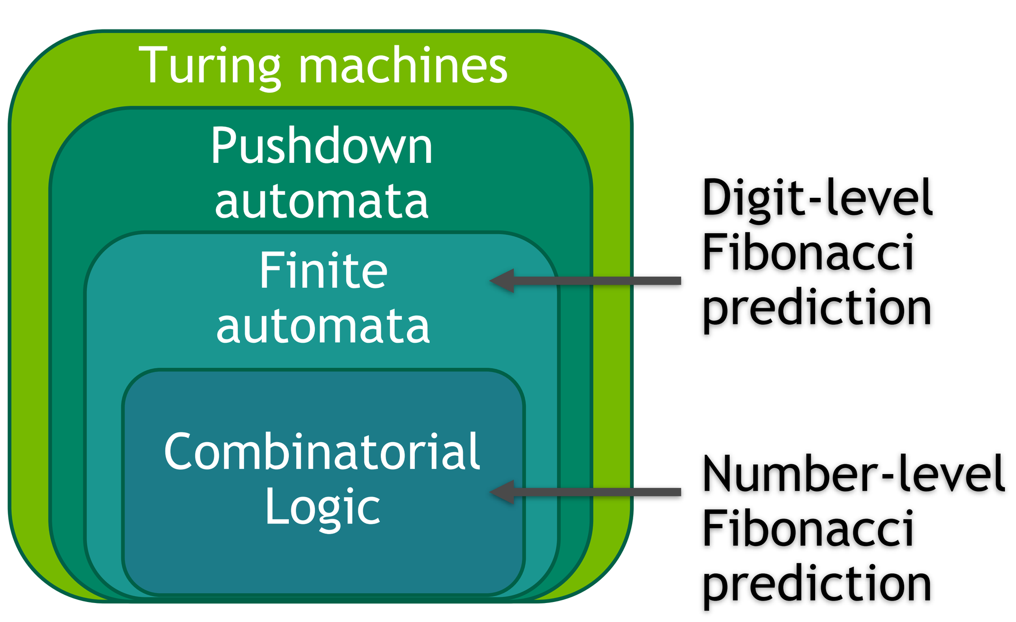
Digit-level (RNN)



A digit input per a time step

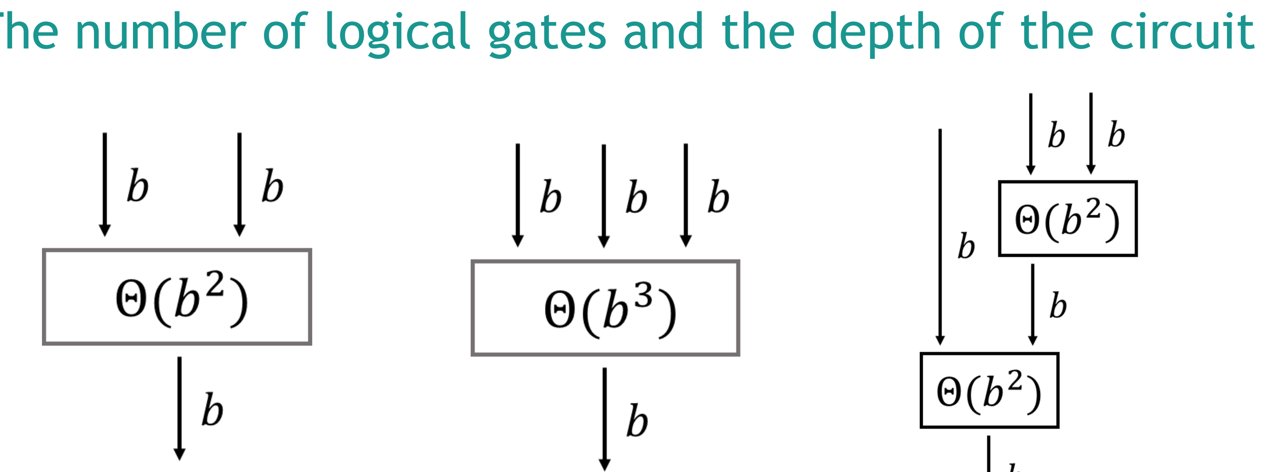
Predict the **digits sequentially**

COMPUTATIONAL POWERS



DIFFICULTY AND COMPLEXITY

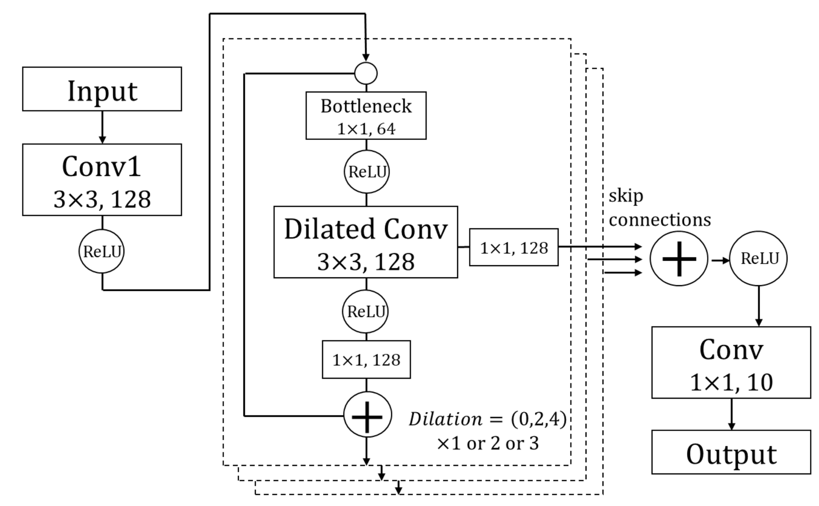
The number of logical gates and the depth of the circuit



Order-2	Order-3	Order-2
Width = $\theta(b^2)$	Width = $\theta(b^3)$	Width = $\theta(b^2)$
Depth = 1	Depth = 1	Depth = 2

NUMBER-LEVEL SEQUENCE PREDICTION

Models



Sequences

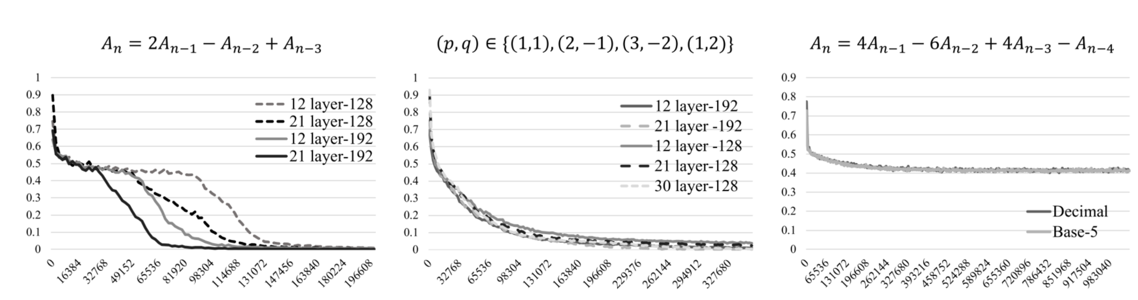
Order-2 Relations

- Fibonacci: $A_{n+2} = A_{n+1} + A_n$
- Arithmetic: $A_{n+2} = 2A_{n+1} - A_n$

Order-3 Relations

- Progression: $A_{n+3} = 3A_{n+2} - 3A_{n+1} + A_n$
- Jumping Fibonacci: $A_{n+3} = A_{n+2} + A_n$

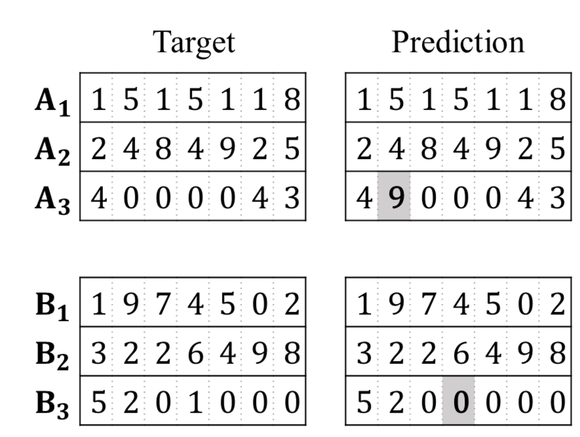
Results



Y-axis: error rates / X-axis: # of training examples

Depth is a better indicator for the complexity

CNNs tend to learn deep but narrow rules

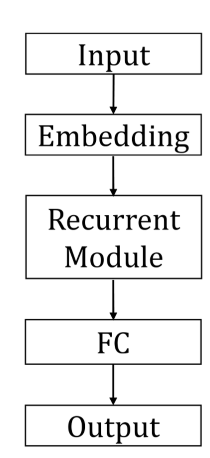


Error examples from number-level Fibonacci prediction

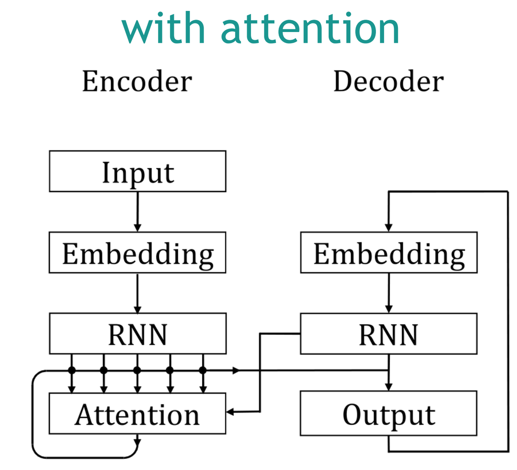
DIGIT-LEVEL SEQUENCE PREDICTION

Models

Recurrent model



Encoder-decoder model with attention



Sequences

Counting sequences ($A_{n+1} = A_n + c$)

- **Finite** automata

Palindromes (e.g. abcd_dcba)

- **Pushdown** automata

Fibonacci/Arithmetic/Geometric

- **Queue** automata (= Turing)

Results

Tasks	Reverse-order (training)	Geometric	Arithmetic	Fibonacci
LSTM	28.4% (1.2%)	79.4%	77.1%	80.5%
GRU	51.9% (0.9%)	69.0%	77.1%	79.3%
Attention(unidirectional)	42.0% (8.8%)	62.8%	77.0%	69.3%
Attention(bidirectional)	0.0% (0.0%)	51.0%	72.9%	60.9%
Stack-RNN	**0.0%** (0.0%)	64.1%	63.8%	69.4%
NTM	**0.0%** (0.0%)	57.1%	65.7%	68.1%

- Reverse-order (palindrome) training errors suggest that RNNs can simulate **finite automata**
- **Memory-augmented models** could simulate up to **pushdown automata**

TAKEAWAYS

- **Number sequence predictions** effectively evaluate computational powers of neural networks
- **Complexity** of a number-level problem can be defined with the combinatorial logic
- **Computational powers** of current recurrent models are **limited** up to those of pushdown automata